Theorem 7. *Every open ball is an open set. Every closed ball is a closed set, where a closed ball is defined to be a set of the form* $\{ p : p$ *is a point and* $d(x,p) \le \varepsilon \}$.

Theorem 8. *Let* A *and* B *be closed sets. Then* $A \cap B$ *and* $A \cup B$ *are closed sets.*

Theorem 9. *Let* A *and* B *be open sets. Then* $A \cap B$ *and* $A \cup B$ *are open sets.*

1.5 Notations for some sets. We will adopt the following notation for this class (most of which are quite standard). The notation $x \in A$ means that the point x is a member, or element, of the set A. The symbol $\mathbb{Z}$ denotes the set of all integers ($\mathbb{Z} = \{0, 1, -1, 2, -2, \dots\}$), $\mathbb{N}$ denotes the strictly positive integers ($\mathbb{N} = \{1, 2, 3, \dots\}$) and ω denotes the the non-negative integers ($\omega = \mathbb{N} \cup \{0\}$). Also, $\mathbb{R}$ denotes the set of all real numbers (we will say more later about what the term "real number" really means), $\mathbb{Q}$ denotes the set of all rational numbers ($\mathbb{Q} = \{ x \in \mathbb{R} : x = n/m$ for some $n, m \in \mathbb{Z} \}$), and $\mathbb{P}$ denotes the set of all irrational numbers ($\mathbb{P} = \{ x \in \mathbb{R} : x \notin \mathbb{Q} \}$).

1.6 Definition. Let A_1, A_2, A_3, $\dots$ be an infinite sequence of sets. Then the infinite union and the infinite intersection of this sequence are defined by:

$$\bigcup_{n=1}^{\infty} A_n \stackrel{\text{def}}{=} \{ x : x \in A_n \text{ for some } n \in \mathbb{N} \} \qquad \bigcap_{n=1}^{\infty} A_n \stackrel{\text{def}}{=} \{ x : x \in A_n \text{ for every } n \in \mathbb{N} \}$$

1.7 Definition. A set A is *bounded* means that there is some point x and some number $r > 0$ such that $A \subset B_r(x)$.

Proposition 10. *Let* A_1, A_2, $\dots$ *be an infinite sequence of closed sets. Then: (a)* $\bigcup_{n=1}^{\infty} A_n$ *is a closed set, and (b)* $\bigcap_{n=1}^{\infty} A_n$ *is a closed set.*

Proposition 11. *Let* A_1, A_2, $\dots$ *be an infinite sequence of open sets. Then: (a)* $\bigcup_{n=1}^{\infty} A_n$ *is an open set, and (b)* $\bigcap_{n=1}^{\infty} A_n$ *is an open set.*

Proposition 12. *Let* A_1, A_2, $\dots$ *be an infinite sequence of non-empty closed and bounded sets. Then* $\bigcap_{n=1}^{\infty} A_n \neq \varnothing$. *Furthermore, the condition that the sets are bounded is necessary.*

Proposition 13 (Bruggink's proposition). *Let* A *and* B *be sets with* $A \subset B$. *Then* $L(A) \subset L(B)$.