Andrew J. Hetzel

# *Statement of Research Interests*

I am an algebraist working primarily in commutative ring theory with secondary interests in certain topics from each of number theory and matrix theory. My current research program can be divided into three general areas: (1) probabilistic questions posed in algebraic contexts, (2) how algebraic structure is impacted by the quantity of the relevant mathematical objects, and (3) multiplicative ideal theory with a particular emphasis on the study of prime ideals.

The fascination of probabilistic questions posed with respect to algebraic phenomena is highlighted by the well-known fact that if a group is more than five-eighths commutative, then the group must be abelian, that is, completely commutative. My joint research with David E. Dobbs has yielded probabilistic-type results in the more number-theoretic genre of so-called "Egyptian fraction expansions" of rational numbers [see "Ahmes Expansions of Rational Numbers of Length Two", Internat. J. Math. Ed. Sci. Tech. **34** (2003) and **"**On Sums of Two Distinct Unit Fractions with Polynomial Denominators", Focus on Commutative Rings Research (2006)]. In addition, I have been involved in some recent work towards developing probabilistic-type results for the diagonalizability of certain matrices [see "The Probability That a Matrix of Integers Is Diagonalizable" with Jay S. Liew and Kent E. Morrison, Amer. Math. Monthly **114** (2007)]. Moreover, this research had its genesis in an undergraduate research project that I led back in 2003. Perhaps the most interesting result developed from this work is the fact that, in a sense, the probability that a 2 by 2 matrix of integers is diagonalizable over the real numbers is exactly 49/72.

The second major facet to my research program is an investigation as to how algebraic structure is affected by the quantity of the relevant mathematical objects. One of the most famous facts along these lines is Wedderburn's theorem that any finite division ring must be a field. Some of these explorations have formed the basis of Rebeca V. Lufi's (nee Lewis) master's thesis on semirings that I led in 2007. I have also recently co-authored a paper ["Going-down Implies Generalized Going-down", Rocky Mountain J. Math. **35** (2005)] with David E. Dobbs where we demonstrated that a certain phenomenon predicated on a finite number of prime ideals is, in fact, equivalent to the corresponding phenomenon predicated on a possibly infinite number of prime ideals.

My third area of research interest is multiplicative ideal theory with an emphasis on the study of prime ideals. It is often the case that information about prime ideals can provide information about arbitrary ideals in a ring in the same way that prime numbers can provide information about arbitrary numbers. For example, I. S. Cohen proved that a commutative ring with $1 \neq 0$ is a Noetherian ring if and only if every prime ideal of the ring is finitely generated. My joint research into prime ideals with A. Serpil Saydam has produced the papers "On the Ascent of ACCP to Simple Overrings" [Comm. Algebra **33** (2005)] and "On the Ascent of Properties Related to Unique Factorization Domains" [Comm. Algebra **34** (2006)]. More recently, I have collaborated with Jonathan A. Cox to create a new generalization of "prime ideal" in the paper "Uniformly Primary Ideals" [J. Pure Appl. Algebra **212** (2008)].