Tennessee Technological University
Mathematics Department

MATH 4750/5750: Category Theory of Sets

I. COURSE DESCRIPTION FROM CATALOG:

Abstracts sets and mappings, categories, sums, universal property, monomorphisms and parts, finite inverse limits, colimits, epimorphisms, the Axiom of Choice, mapping sets and exponentials, covariant and contravariant functoriality of function spaces, Cantor's diagonal argument, powers sets, variable sets, models of additional variation, selected applications. Lec. 3. Cr. 3.

II. PREREQUISITE(S):

C or better in MATH 3400 (or consent of instructor for MATH 5750)

III. COURSE OBJECTIVE(S):

Advanced undergraduate or beginning graduate students need a unified foundation for their study of geometry, analysis, and algebra. This is an introductory course in category theory of abstract sets and mappings for mathematically advanced undergraduates and beginning graduate students in mathematics, computer science, physics, or computer engineering. Its purpose is to integrate a typical undergraduate experience of mathematics major in theoretical mathematics. Thus, the course can serve as an exit course for mathematics majors, or, as a preparatory course for further study and research in mathematics, computer science, or theoretical physics at the graduate level. Computer science students (if present) will be introduced to selected examples of applications of the category theory to computer science.

Graduate students will be assigned additional readings on more advanced topics and applications chosen from the references listed at the end. They will be asked to present these topics in class.

IV. TOPICS TO BE COVERED:

1 Abstract Sets and Mappings
- 1.1 Sets, Mappings, and Composition
- 1.2 Listings, Properties, and Elements
- 1.3 Surjective and Injective Mappings
- 1.4 Associativity and Categories
- 1.5 Separators and the Empty Set
- 1.6 Generalized Elements
- 1.7 Mappings as Properties
- 1.8 Additional Exercises

2 Sums, Monomorphisms, and Parts
- 2.1 Sum as a Universal Property
- 2.2 Monomorphisms and Parts
- 2.3 Inclusion and Membership

Students with a disability requiring accommodations should contact the Office of Disability Services (ODS). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The ODS is located in the Roaden University Center, Room 112; phone 372-6119.

Last Revised: 06/07/05

8.5 Additional Exercises

9 Introduction to Variable Sets (Optional)
- 9.1 The Axiom of Infinity: Number Theory
- 9.2 Recursion
- 9.3 Arithmetic of *N*
- 9.4 Additional Exercises

10 Models of Additional Variation (Optional)
- 10.1 Monoids, Posets, and Grupoids
- 10.2 Actions
- 10.3 Reversible Graphs
- 10.4 Chaotic Graphs
- 10.5 Feedback and Control
- 10.6 To and From Idempotents
- 10.7 Additional Exercises

11 Selected Application and Examples from Computer Science (Optional, see [5] and references therein)
- 11.1 Omega-Algebras
- 11.2 Functional Programming Language (FPL) category
- 11.3 Exponentiation, eval, and curry
- 11.4 Functors: List, maplist.
- 11.5 F-Algebras and F-homomorphisms
- 11.6 Adjoints
- 11.7 Cartesian Closed Categories
- 11.8 Implicit Conversions and Generic Operators
- 11.9 Programming Language Semantics
- 11.10 Recursive Domain Equations

V. ADDITIONAL INFORMATION:

Recommended prerequisite: One or more of the following: MATH 3510, 3520, 4050/5050, 4110/4120, 5110/51120, 4530/4540, 5530/5540.

Graduate credit is earned on the basis of additional work that can be required by the instructor per TTU Graduate Catalog] Graduate students will be expected to present topics and applications in class.

The main body of the course consists of Chapters 1 – 5 from [1]. Chapters 7 – 10 from [1] can be covered in succession depending on the interests of students and the instructor. Computer applications can be selected from [5] and references therein.

VI. POSSIBLE TEXTS AND REFERENCES:

1. F. William Lawvere, Robert Rosebrugh, W. Lawvere, R. Rosebrugh, “Sets for Mathematics”, Cambridge University Press, 2002 (ISBN 0-521-80444-2, 0-521-01060-8 pbk).
2. F. William Lawvere, Stephen Hoel Schanuel, “Conceptual Mathematics : A First Introduction to Categories”, Cambridge University Press, 1997
3. Saunders Mac Lane, “Categories for the Working Mathematician”, Springer-Verlag, 1971.
4. George Glatzer, “Universal Algebra”, Van Nostrand, 1968.
5. Benjamin C. Pierce, “Basic Category Theory for Computer Scientists”, Foundations of Computing Series, The MIT Press, Cambridge, MA, 1991 [ISBN 0-262-66071-7]
6. Andrea Asperti and Giuseppe Longo, “Categories, Types, and Structures: An Introduction to Category Theory for the Working Computer Scientists.” The MIT Press, Cambridge, MA, 1991.
7. Michael Barr and Charles Wells, “Category Theory for Computing Science”, Prentice Hall, 1990.

VII. ANY TECHNOLOGY THAT MAY BE USED: