

Tennessee Technological University
Mathematics Department

MATH 4750/5750: Category Theory of Sets

I. COURSE DESCRIPTION FROM CATALOG:

Abstracts sets and mappings, categories, sums, universal property, monomorphisms and parts, finite inverse limits, colimits, epimorphisms, the Axiom of Choice, mapping sets and exponentials, covariant and contravariant functoriality of function spaces, Cantor's diagonal argument, powers sets, variable sets, models of additional variation, selected applications. Lec. 3. Cr. 3.

II. PREREQUISITE(S):

C or better in MATH 3400 (or consent of instructor for MATH 5750)

III. COURSE OBJECTIVE(S):

Advanced undergraduate or beginning graduate students need a unified foundation for their study of geometry, analysis, and algebra. This is an introductory course in category theory of abstract sets and mappings for mathematically advanced undergraduates and beginning graduate students in mathematics, computer science, physics, or computer engineering. Its purpose is to integrate a typical undergraduate experience of mathematics major in theoretical mathematics. Thus, the course can serve as an exit course for mathematics majors, or, as a preparatory course for further study and research in mathematics, computer science, or theoretical physics at the graduate level. Computer science students (if present) will be introduced to selected examples of applications of the category theory to computer science.

Graduate students will be assigned additional readings on more advanced topics and applications chosen from the references listed at the end. They will be asked to present these topics in class.

IV. TOPICS TO BE COVERED:

- 1 Abstract Sets and Mappings
 - 1.1 Sets, Mappings, and Composition
 - 1.2 Listings, Properties, and Elements
 - 1.3 Surjective and Injective Mappings
 - 1.4 Associativity and Categories
 - 1.5 Separators and the Empty Set
 - 1.6 Generalized Elements
 - 1.7 Mappings as Properties
 - 1.8 Additional Exercises

- 2 Sums, Monomorphisms, and Parts
 - 2.1 Sum as a Universal Property
 - 2.2 Monomorphisms and Parts
 - 2.3 Inclusion and Membership

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- 2.4 Characteristic Functions
- 2.5 Inverse Image of a Part
- 2.6 Additional Exercises

- 3 Finite Inverse Limits
 - 3.1 Retractions
 - 3.2 Isomorphism and Dedekind Finiteness
 - 3.3 Cartesian Products and Graphs
 - 3.4 Equalizers
 - 3.5 Fullbacks
 - 3.6 Inverse Limits
 - 3.7 Additional Exercises

- 4 Colimits, Epimorphisms, and the Axiom of Choice
 - 4.1 Colimits are Dual to Limits Sections
 - 4.2 Epimorphisms and Split Surjections
 - 4.3 The Axiom of Choice
 - 4.4 Partitions and Equivalence Relations
 - 4.5 Split Images
 - 4.6 The Axiom of Choice as the Distinguishing Property of Constant/Random Sets
 - 4.7 Additional Exercises

- 5 Mapping Sets and Exponentials
 - 5.1 Natural Bijection and Functoriality
 - 5.2 Exponentiation
 - 5.3 Functoriality of Function Spaces
 - 5.4 Additional Exercises

- 6 Summary of the Axioms and an Example of Variable Sets
 - 6.1 Axioms for Abstract Sets and Mappings
 - 6.2 Truth Values for Two-Stage Variable Sets
 - 6.3 Additional Exercises

- 7 Consequences and Uses of Exponentials (Optional)
 - 7.1 Concrete Duality: The Behavior of Monics and Epics under Contravariant Functoriality of Exponentiation
 - 7.2 The Distributive Law
 - 7.3 Cantor's Diagonal Argument
 - 7.4 Additional Exercises

- 8 More on Power Sets (Optional)
 - 8.1 Images
 - 8.2 The Covariant Power Set Functor
 - 8.3 The Natural map $PX \rightarrow 2^{2X}$
 - 8.4 Measuring, Averaging, and Winning with V-Valued Quantities

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8.5 Additional Exercises

9 Introduction to Variable Sets (Optional)

9.1 The Axiom of Infinity: Number Theory

9.2 Recursion

9.3 Arithmetic of N

9.4 Additional Exercises

10 Models of Additional Variation (Optional)

10.1 Monoids, Posets, and Grupoids

10.2 Actions

10.3 Reversible Graphs

10.4 Chaotic Graphs

10.5 Feedback and Control

10.6 To and From Idempotents

10.7 Additional Exercises

11 Selected Application and Examples from Computer Science (Optional, see [5] and references therein)

11.1 Omega-Algebras

11.2 Functional Programming Language (FPL) category

11.3 Exponentiation, eval, and curry

11.4 Functors: List, maplist.

11.5 F-Algebras and F-homomorphisms

11.6 Adjoints

11.7 Cartesian Closed Categories

11.8 Implicit Conversions and Generic Operators

11.9 Programming Language Semantics

11.10 Recursive Domain Equations

V. ADDITIONAL INFORMATION:

Recommended prerequisite: One or more of the following: MATH 3510, 3520, 4050/5050, 4110/4120, 5110/51120, 4530/4540, 5530/5540.

Graduate credit is earned on the basis of additional work that can be required by the instructor per [TTU Graduate Catalog]. Graduate students will be expected to present topics and applications in class.

The main body of the course consists of Chapters 1 – 5 from [1]. Chapters 7 – 10 from [1] can be covered in succession depending on the interests of students and the instructor. Computer applications can be selected from [5] and references therein.

VI. POSSIBLE TEXTS AND REFERENCES:

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1. F. William Lawvere, Robert Rosebrugh, W. Lawvere, R. Rosebrugh, "Sets for Mathematics", Cambridge University Press, 2002 (ISBN 0-521-80444-2, 0-521-01060-8 pbk).
2. F. William Lawvere, Stephen Hoel Schanuel, "Conceptual Mathematics : A First Introduction to Categories", Cambridge University Press, 1997
3. Saunders Mac Lane, "Categories for the Working Mathematician", Springer-Verlag, 1971.
4. George Glatzer, "Universal Algebra", Van Nostrand, 1968.
5. Benjamin C. Pierce, "Basic Category Theory for Computer Scientists", Foundations of Computing Series, The MIT Press, Cambridge, MA, 1991 [ISBN 0-262-66071-7]
6. Andrea Asperti and Giuseppe Longo, "Categories, Types, and Structures: An Introduction to Category Theory for the Working Computer Scientists." The MIT Press, Cambridge, MA, 1991.
7. Michael Barr and Charles Wells, "Category Theory for Computing Science", Prentice Hall, 1990.

VII. ANY TECHNOLOGY THAT MAY BE USED:

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